Chemical Equilibration
in Relativistic Heavy Ion Collisions

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Abstract

In the hadronic sector of relativistic heavy ion physics, the $\rho \leftrightarrows 2\pi$ reaction is the strongest one, strong enough to equilibrate the ρ with the pions throughout the region from chemical freezeout to thermal freezeout when free-particle interactions (with no medium-dependent effects) are employed. Above the chiral restoration temperature, only ρ's and π's are present, in that the chirally restored A_1 is equivalent to the ρ and the mesons have an SU(4) symmetry, with no dependence on isospin and negligible dependence on spin. In the same sense the σ and π are "equivalent" scalars. Thus the chirally restored $\rho \leftrightarrows 2\pi$ exhaust the interspecies transitions. We evaluate this reaction at T_c and find it to be much larger than below T_c, certainly strong enough to equilibrate the chirally restored mesons just above T_c. When emitted just below T_c the mesons remain in the $T_c + \epsilon$ freezeout distribution, at least in the chiral limit because of the Harada-Yamawaki "vector manifestation" that requires that mesonic coupling constants go to zero (in the chiral limit) as T goes to T_c from below. Our estimates in the chiral limit give deviations in some particle ratios from the standard scenario (of equilibrium in the hadronic sector just below T_c) of about double those indicated experimentally. This may be due to the neglect of explicit chiral symmetry breaking in our estimates. We also show that the instanton molecules present above T_c are the giant multipole vibrations found by Asakawa, Hatsuda and Nakahara and of Wetzorke et al. in lattice gauge calculations. Thus, the matter formed by RHIC can equivalently be called: chirally restored mesons, instanton molecules, or giant collective vibrations. It is a strongly interacting liquid.

1 Introduction

One of the recurring results from relativistic heavy ion collisions, emphasized by Braun-Munzinger and Stachel and collaborators, found first in AGS and then at CERN energies and most recently at RHIC [1] has been the high degree of chemical equilibration of the hadronic products of the collisions. All of them, with exceptions we shall discuss, freeze out in chemical composition at the same temperature, essentially the temperature for chiral restoration T_c.

Many attempts to explain this equilibration in the accepted scenario of the quark gluon plasma(QGP): increased equilibration because of the higher number of degrees of freedom in the QGP, or elsewhere, in the greater number of degrees of the high temperature hadron gas because of dropping masses, were ultimately defeated because of the small coupling constants in these phases. In the QGP the perturbative coupling constants were simply too small to bring about equilibration. Efforts in the hadronic sector were defeated by the RG results of Harada and Yamawaki [2] that hadronic coupling constants go to zero at T_c in the chiral limit.

Recently Brown et al.[3] have shown that a second-order phase transition can be constructed for RHIC physics (ignoring the small baryon number in central collisions) by making the chirally restored π and σ massless just above T_c. In this way there is a continuity with the massless π and σ, in the chiral limit, just below T_c. Quark masses [1], measured in quenched lattice gauge studies [4] are $\gtrsim 1$ GeV both at $\frac{3}{2}T_c$ and $3T_c$. We shall assume them to have this value [2] right down to T_c. In BLRS[3] at T_c, the color Coulomb interaction brought the π- and σ-masses down from 2 GeV to 1.5 GeV, and then the 4-point instanton molecule interaction brought them the rest of the way to zero.

The four-point interaction [3] acted as a driving force in an RPA, with all bound $\bar{q}q$ states at unperturbed energy 1.5 GeV in a Furry representation (Coulomb eigenstates); i.e., quarkonium. Therefore Brown's [5] degenerate schematic model, in which eigenfunctions, etc., have simple analytic forms, could be used.

It was pointed out that the resulting collective excitations, the giant multipole states, were seen in the Asakawa et al.[6] LGS, also by Wetzorke et al.[7].

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[1] The "quark mass" we refer to in this paper is the chirally invariant thermal mass, not the current quark mass that breaks chiral symmetry.
[2] As noted in BLRS[3] with our dynamic confinement the mass measured by the Polyakov loop is never reached.
[3] The 4-point interaction is modelled as a δ-function with constant coefficient; therefore, trivially factorizable.

An important new development was the argument by Braun-Munzinger et al. [8] that hadron multiplicities in central high-energy nucleus-nucleus collisions are established essentially at the phase boundary between chirally broken and chirally restored matter. This must result from multiparticle collisions, which are particularly strong just above T_c.

In this note we develop the schematic model to include nonlinear couplings of the giant vibrations just above T_c. We study, in particular, the $\rho \to 2\pi$ reaction in the chirally restored region, both the ρ and π being giant resonances. This way of looking at the ρ and π was developed long ago as the loop sum in the Nambu-Jona-Lasinio approach in the chirally broken region of hadrons, but it is much simpler here because of the degeneracy in Coulomb-bound $\bar{q}q$ states above T_c. We note that the chirally restored π's and σ's are the same entities, also the chirally restored vectors and axial vectors. Therefore, we need consider only the interaction $\rho \leftrightarrows 2\pi$ to determine the nonlinearity of the vibrations, this covering all possibilities.

2 The Free ρ and π as Giant Resonances; The Strong $\rho \to 2\pi$ Transition as a Nonlinearity in the Vibrations

Many research workers are not used to thinking of the ρ-meson as a giant dipole resonance, but that will be a convenient place to begin. The point is that the ρ, all by itself, is a many-body problem in the Nambu-Jona-Lasinio language. It is a giant dipole vibration of the negative energy sea, a sum of quark-antiquark pieces.

In NJL at zero density and temperature the negative energy states are filled up to a cutoff $\Lambda \sim 700$ MeV. In the modern way of looking at matters, this Λ is the Wilsonian matching scale for constituent quarks[9]. The ρ or π are then obtained as a sum of loop diagrams, the vertex function Γ equal to γ_μ or γ_5 for ρ and π, respectively, as in random phase approximation, as shown in Fig. 1, except that in the chirally broken sector the Coulomb interaction is unimportant. The backward going lines represent holes in the negative energy sea, the forward-going ones, particles.

A good complete review of these matters is given by Vogel & Weise [10].

Now the width for the free ρ to decay into two pions is large, ~ 150 MeV, meaning that the lifetime of the ρ is $\tau \sim 4/3$ fm/c, still long enough that the ρ can be distinguished as a real particle. The point we wish to make is that the $\rho \to 2\pi$ transition in medium can be considered to be a nonlinearity, a giant vector vibration changing into two giant pseudoscalar ones. This nonlinear coupling will damp the ρ vibrations, mixing the energy between ρ and π ones,
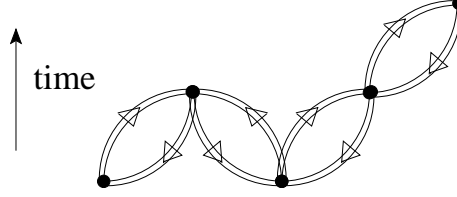
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Fig. 1. The sum of bubbles in NJL, which can go either forward or backward in time, the latter representing ground state correlations. The solid dots represent the vertex function, γ_μ for the ρ, γ_5 for the π, multiplied by a δ-function and coupling constant. The double lines indicate that quark and antiquark, or quark and quark holes, are Coulomb bound states; i.e., are in the Furry representation, which we will go over to above T_c.

essentially powering the many-body interactions which give the collectivity to these vibrations.

Note that the $\rho \rightarrow 2\pi$ transition is already a strong one in the broken symmetry sector. Also the $A_1 \rightarrow \rho + \pi$ has an even larger width, and can be considered the same way as a collective excitation. However, the enhancement due to collective effects, although large, is not nearly as large as it would be if the unperturbed energy of all of the particle-antiparticle loops were the same, as we shall find to be the case in the chirally restored sector.

3 Coulomb Bound States Above T_c: The Furry Representation

Shuryak & Zahed [11] introduced Coulomb bound states of quark and antiquark, or of quark-particle and quark-hole for the region of initial RHIC temperatures, suggesting that the breakup of these molecules would help with the observed early equilibration. Below $T \sim 2T_c$ the $\bar{q}q$ states would be bound, quite strongly as T moves down towards T_c. The color Coulomb interactions can be summed to all orders by going over to quarkonium which has quark and antiquark in Coulomb bound states. This is the so called Furry representation, well known in atomic physics, and shown by the double lines in Fig. 1. This gives us a convenient starting point.

The cutoff $\Lambda_{\mathrm{NJL}} \sim 700$ MeV may be considered as a sort of order-parameter for the constituent quarks, as well as Wilsonian matching scale, in that the negative energy quarks comprise the condensate which breaks chiral symmetry in giving hadron masses. The chiral symmetry breaking scale for hadrons is $\Lambda_{\chi\mathrm{SB}} = \sqrt{2}\Lambda_{\mathrm{NJL}} \sim 1$ GeV [2]. As T moves above T_c the (collective) chiral symmetry breaking state disappears. The chirally restored zero-mass π, σ and ρ become the thermodynamic variables just above T_c. This means that the scale drops from $\Lambda_{\chi SB} \sim 4\pi f_\pi \sim 1$ GeV towards the infrared, the scale just above T_c being zero. The large distance color Coulomb coupling constant just
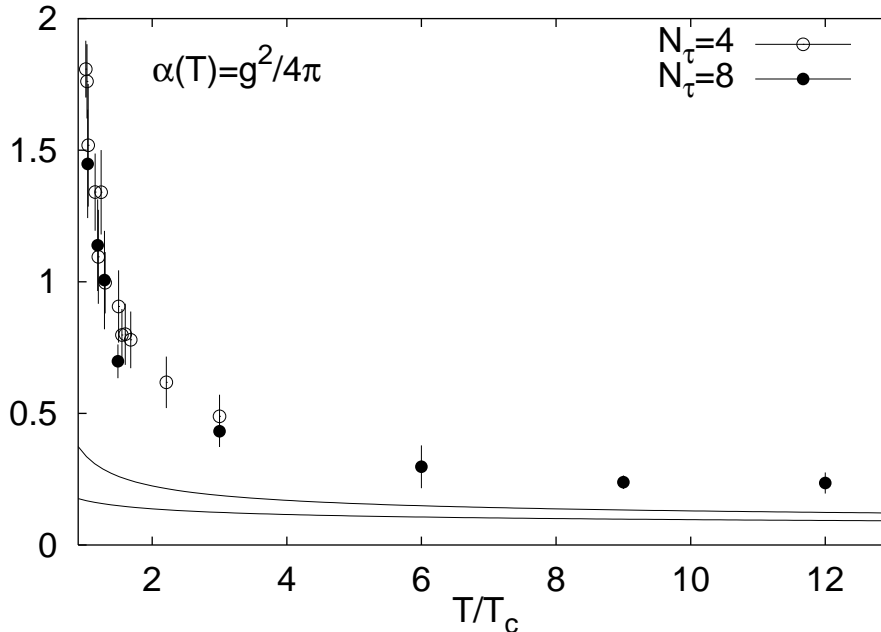
Fig. 2. The large distance behavior of $\alpha_S(T)$ from evolution of the Polyakov loop in quenched LGS[12].

above T_c has been evaluated by F. Zantow [12] from the Polyakov loops in quenched LGS. We show the behavior of $\alpha(T)$ in Fig. 2 for $T \gtrsim T_c$.

The Bielefeld $\alpha_s(T)$ does not have the Casimir 4/3 in it that we use. Furthermore the α_s is calculated with heavy quarks. In the case of light quarks at T_c, addition of the velocity-velocity interaction in the appropriate helicity states doubles the effective Coulomb interaction. Thus our effective large-distance $\alpha_s(T)$ would reach a value of $\sim 16/3$ at T_c, giving a $g_{eff} \sim 8$ for the color Coulomb interaction. This is a strong indication of the QCD scale having moved far towards the infrared, indicating very low mass modes as thermodynamic variables.

This large distance behavior does not give us the effective color Coulomb interaction to be used in calculating our π, σ, ρ bound states, which are small in extent. For these we need the $\alpha_s(T_c, r)$ for small r. This is quite complicated because we know that $\alpha_s(T, r)$ goes to zero as $r \to 0$.

We received from Olaf Kaczmarek [13] results in full (unquenched) QCD. The results for the free energy, color singlet internal energy, etc. are very nearly the same as in quenched QCD, except that the T_c (quenched) is rescaled to T_c (unquenched). We shall assume this to be generally true in what follows, although the vibrations have not yet been calculated in full QCD. Note that this is full QCD for heavy quarks, so we must add the additional effects for light quarks, such as the velocity-velocity interaction and the instanton molecule

5

interaction, which would not enter into the heavy-quark sector.

We can, however, as in BLRS[3], use the parameter α_s obtained from the binding of charmonium above T_c, namely $\alpha_s \simeq 0.5$. The charmonium atoms have about the same radius $r \gtrsim 1/3$ fm as our σ, π, ρ at T_c. With $\alpha_s = 0.5$, doubled to take into account the velocity-velocity interaction together with the Nambu-Jona-Lasinio interaction extended up into the chirally restored region as done in BGLR[14], BLRS obtained zero energy for their π and σ. Below we discuss how the ρ mass is brought to zero at T_c in the chiral limit.

We remark that very strong coupling is clearly needed at $T = T_c + \epsilon$ because the ~ 2 GeV unperturbed energy of the quark and antiquark must be brought to zero by the binding potentials in the chiral limit.

As discussed by BLRS[3], the instanton molecule interactions are expected to be unimportant by a temperature $T \sim 2T_c$. Note that in the chirally restored region, the scalar and pseudoscalar degrees of freedom should be the same, as well as those of the vector and axial vector. Thus, one can include all excitations within the framework of the π and the ρ, as we shall do. Further note that in accord with Asakawa et al., BLRS found the spin dependence to be negligible.

4 The $\rho \leftrightarrows 2\pi$ Reaction as a Coupling Between Coulomb Bound States.

Ultimately we hope to be able to construct the coupling between giant resonances in the chirally restored region, but as a first step will include only the color Coulomb interaction, which leaves out the many-particle aspect of the giant resonances. As noted, in the chirally restored region we need consider only ρ's and π's, which exhaust the chirally restored degrees of freedom.

In Fig. 3 we show how a $\bar{q}q$ bound state with the quantum numbers of the ρ can decay into two pions.

For the transition matrix element (the matrix element to be put into Fermi's golden rule) we have

$$M = \langle \bar{q}\gamma_5 q(1)\bar{q}\gamma_5 q(2)\delta H_{23}\bar{q}\gamma_\mu q(3)\rangle \tag{1}$$

where the δH_{23} is the Coulomb interaction

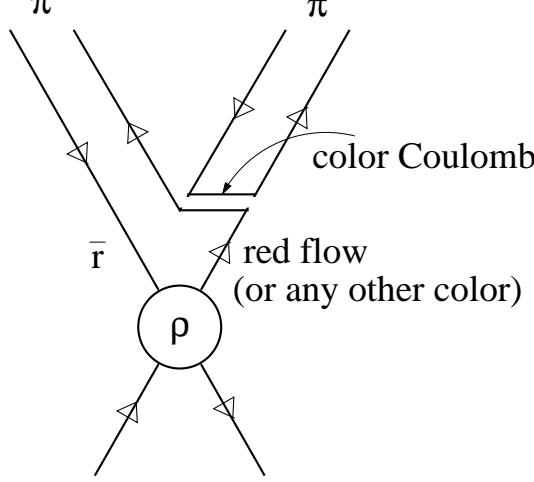$$\delta H = \frac{e^2}{r_{23}} \simeq \frac{e^2}{r} + \frac{e^2 r_3 \cos\theta_1}{r^2} \tag{2}$$

6

Fig. 3. How the color flows to show that we make colorless molecules by color Coulomb exchange. We show only the single color Coulomb interaction which creates an additional quark-antiquark or quark-quark-hole pair. The quark and antiquark lines are all Coulomb states. A factor 2 arises because the color Coulomb interaction could equally well start from the down-going line on the left; another factor of 2 because the two π's could equally contribute as two σ's.

Here r is the distance between the pion at point 2, which is so small that we take it to be a point, and the distributed mass of the ρ-meson with r.m.s. radius $[\langle r_3^2 \rangle]^{1/2} \simeq 0.5$ fm, at point 3. The distance r will be ~ 1 fm, the average distance between particles. We must carry the expansion to obtain the $\cos\theta$ dependence, in order to match to the ρ which we assume to be polarized in the z-direction. [4]

It is of interest to compare the δH of eq. (2) with that of the chirally broken $\rho \to 2\pi$ discussed in Sec. 2. We can obtain this from Ref.[15]. This is particularly simple for massless pions [5] which should not be too bad an approximation;

$$M = g_v m_\rho. \tag{3}$$

The ratio of the relevant part of eq. (2) to eq. (3) is

$$R = \frac{e^2 r_1 \sqrt{1/3}/r^2}{g_v m_\rho} = \frac{4\pi\alpha_s r_1 \sqrt{1/3}/r^2}{g_v m_\rho} \tag{4}$$

where we have replaced $\cos\theta$ by $\sqrt{1/3}$, since the square averaged over Ω will

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[4] In fact, at T_c the ρ polarized in the time direction is most important. It is related to the one in the z-direction by the consistency condition $\partial\rho_\mu/\partial x_\mu = 0$. Thus, one can say that they are the same degree of freedom.

[5] We thank M. Prakash for this observation.

give 1/3.

We should multiply this ratio by 4 because the Coulomb interaction could equally well begin from the antiquark line of the ρ (left-hand side in Fig. 3) and in the chirally restored regime the ρ can go into two σ's as well as two π's, the π and σ being the same in the chirally restored sector.

Now, as noted in the Introduction, the radius r_3 of the ρ is ~ 0.5 fm, and we found the effective α_s to be ~ 1. We take $g_v \sim 5$. Since the particles are ~ 1 fm apart we take $r = 1$ fm. Note that we do not have asymptotic states in the many body calculation, so energy conservation will be somewhat blurred. We find $R \sim 0.18$, i.e. the nonlinearity in the $\bar{q}q$ Coulomb bound states is $\sim 2/3$ as large as that of the chirally broken ρ when multiplied by the 4 noted above. However, the latter is a collective vibration, as calculated in NJL, so before we can make a true comparison, we must carry out the same type of loop expansion in the chirally restored region, putting in collective effects.

At T_c, the effective α_s is about unity. As will be described in the next section, adding loops to the $\bar{q}q$ bound states and summing them will increase the process, Fig. 6 by a factor $\sim 4^3 = 64$ at $T = T_c + \epsilon$; i.e., the interaction of the vibrations at $T \gtrsim T_c$ is much greater than the interaction between $\bar{q}q$ Coulomb bound states. Of course, what we have done is to add up an infinite number of attractive contributions in the bubble sum. This is correct in random phase approximation when governed by a symmetry principle; i.e., see § 5 Ch. V of Brown[5] where the spurious translational state in nuclei is brought to zero energy by just such a procedure. The same occurs in the Anderson mode in superconductors or the Goldstone mode $-$ our π-meson $-$ in particle physics, except that these are real, not spurious modes.

However, in the part of the nuclear many-body problem not governed by symmetry principles, the original Kuo-Brown procedure of including only one bubble has turned out to be quite good [16]. Of course our situation here is different from the nuclear many-body problem, in that we have both the Coulomb interaction and bubbles. However, the general idea may be the same. In Fig. 4 we show a typical higher order graph summed by Kirson in his "bubbles in bubbles" translated to our Coulomb+instanton molecule interactions.

The "bubbles in bubbles" increase the contributions of repulsive terms – the process shown in Fig. 4 is an exchange term – in higher order. Once the interactions are more accurately determined; e.g. by LGS, it will be interesting to attempt to sum them. (In the case of the nuclear many-body problem, even though it is a case of strong interaction, such a systematic treatment has been carried out[16].) However, for the moment we will introduce just one bubble, assuming the higher-order bubble terms to be cut down by the exchange terms with bubbles in bubbles.
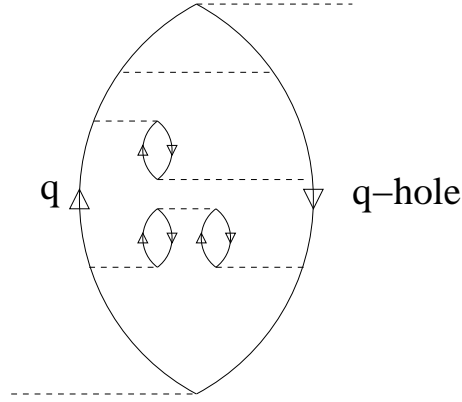
Fig. 4. Higher-order graph. In this figure we have not used Coulomb bound states as representation but bare quark and quark-hole states, in order to make clear the connection with the nuclear many-body problem. The dashed lines are the Coulomb interaction. We can also attach the bubbles to the quark or quark-hole or to each other by instanton molecule interactions.
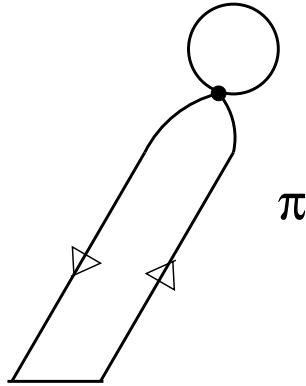

Fig. 5. Adding a bubble to one of the π's in Fig. 3, which also involves adding a propagator. The solid dot is the instanton induced interactions. All lines are Coulomb bound states.

From the development Fig. 5 and following in BLRS[3] we note that adding a bubble as in Fig. 5 will increase the contribution by the factor

$$C = 1 + \frac{1 \text{ GeV } F}{1.5 \text{ GeV}} = \frac{4}{3} \tag{5}$$

where $F \simeq 0.5$ is the correction for nonlocality. (See Eq. (42) and Appendix of BLRS[3]). Now the bubble can be added to either π or the initial ρ in Eq. 3 so that the overall increase is by a factor of $(1 + 1/3)^3 \simeq 2.4$.

Including the factor 2.4 in R we find the Golden rule matrix element M to be 1.6 times that for the chirally broken $\rho \leftrightarrows 2\pi$, or the width to be a factor of $(1.6)^2$ larger, i.e.

$$\Gamma = 380 \text{ MeV}. \tag{6}$$

Whereas the above is hardly a quantitative calculation, we do end up with a width Γ somewhere in between the m_ρ of 560 MeV of BLRS and the 280 MeV we shall end up with here. This is satisfying in that in strong coupling the width of a particle should end up more or less equal to its mass.

In order to come down to the one-bubble insertion as taking care of the role of vibrations in the nuclear many-body problem, a lot of calculations, ending in Kirson's work[16] had to be carried out. In our present situation we have both color Coulomb, which scales rapidly with energy, and the instanton molecule interaction which is tied to the hard glue. Thus a quantitative many-body calculation would be difficult. We do, however, have the lattice calculations. Dealing with heavy quarks these do not include the velocity-velocity and instanton molecule interactions, but this may be an advantage for investigating how the widths of the resonances grow as T moves downwards towards T_c, in that they should not get out of hand, which they do with our estimates including everything.

Given the fact that the coupling must be so strong as to bring the unperturbed (chirally invariant) mass $2m_q$ down to zero in order to construct the π and σ in the chiral limit, we believe that they will give a strong $\rho \leftrightarrows 2\pi$ transition. Since we have only ρ's and π's in the chirally restored region above T_c, this will result in complete equilibration. (It should be noted that below – but away from – T_c the $\rho \leftrightarrows 2\pi$ reaction is the strongest, and results in equilibration of the ρ with the two pions down to thermal freezeout.)

We finish this section by showing that the same sort of strong interactions that produced the $\rho \leftrightarrows 2\pi$ nonlinearities will give large meson scattering cross sections. We draw the color flow in Fig. 6 for pion-pion scattering. In the case of the pions, we would use the first term e^2/r of δH, eq. (2) and it is easy to see that we will get cross section of hadronic size, since $\alpha_s \sim 1$ at T_c.

5   Constructing Vibrations by connecting Coulomb Bound States

Now the instanton molecule interaction for $T \gtrsim T_c$ is sizable, and effects in producing giant collective modes are maximized by the degeneracy in unperturbed energies at the common 1.5 GeV energy of the Coulomb bound states. Because of the degeneracy in unperturbed state energy, the problem for $T > T_c$ actually resembles more that of the giant dipole state in nuclei [17] than NJL for $T < T_c$. From Ref.[5] one can see that if one includes only loops going forward in time, then the energy of the giant collective mode is brought down with the entire trace of the secular matrix for these states. The
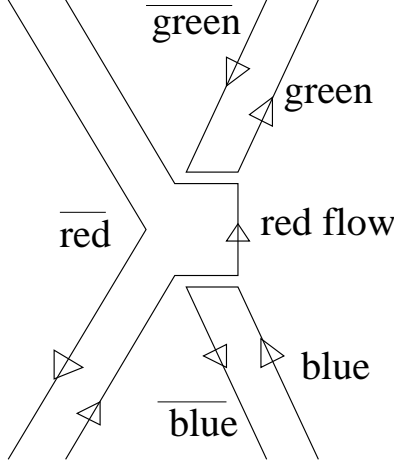
Fig. 6. Pion-pion scattering in the Furry representation. The red-$\overline{\text{blue}}$ and green-$\overline{\text{red}}$ contributions to the color Coulomb interaction are shown; of course, the color must be summed over. The other color Coulomb (singlet) interactions which establish the Furry representation are not shown.

argument is repeated in Appendix A. Furthermore, if the forward going loops bring the energy of the collective mode halfway from the 1.5 GeV unperturbed energy to 0.75 GeV, then including the backward going loops ("ground-state correlations") will bring the energy all of their way to zero[5]. In other words, all of the interactions conspire so as to move the collective mode in energy, leaving the remaining non-collective modes at their unperturbed energy of 1.5 GeV. (As noted earlier, the fact that the pion is a Goldstone mode governs the attraction from all sets of bubbles.)

Indeed, with such strong interactions we will certainly have thermal equilibrium in the region just above T_c (and probably higher).

Although not yet confirmed by LGS,[6] the scenario by Harada & Yamawaki [2] in which the width of the ρ-meson goes to zero (in the chiral limit) as $T \to T_c$ from below, should be helpful in providing equilibrated hadron emission. In their scheme $g_V \to 0$ as $T \to T_c$ from below. In fact, Brown and Rho [9] showed that $g_V^{\star 2}/m_\rho^{\star 2}$ goes smoothly through T_c from LGS of quark number susceptibility. Thus if $m_\rho^\star \to 0$, $g_V^\star$ does also. Furthermore, the G in BGLR[14] is essentially the ratio $g_{\sigma QQ}^2/m_\sigma^2$ and the rough constancy in G means that as m_σ goes to zero at T_c in the chiral limit, so does $g_{\sigma QQ}$.

Thus, the system of highly equilibrated chirally restored mesons just above T_c cut loose at T_c into an environment in which the interactions are zero in the chiral limit just below T_c. This is obviously the optimum condition for chemically equilibrated particle yields at freezeout, which takes place at T_c, as foreseen by Braun-Munzinger et al.[8].

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[6] The necessary unquenched calculations have not been carried out.

6 Construction of $F = E - \tau\sigma$.

The Helmholtz free energy $F = E - \tau\sigma$ enters into the exponent of the Wilson line as $\exp(\beta F)$ and is proportional to the string tension so $F = 0$ at T_c for heavy mesons. In BLRS we found it extremely useful to use the Bielefeld lattice results for heavy mesons in charmonium, and to add the additional effects relevant for light quarks such as those from the Ampere's law velocity-velocity interaction and the instanton molecule interaction. Recently there has been a lot of work at Bielefeld evaluating the Helmholtz free energy for heavy quarks [18]. It is of interest to calculate F for our chirally restored mesons, which add some new aspects, here.

In the appendix we give the solutions for the instanton molecule Lagrangian of BLRS. For any given spin and isospin that the Coulomb bound states are coupled to, they form an unperturbed representation degenerate in energy, each $\bar{q}q$ bound state lying at 1.5 GeV. All matrix elements, both diagonal and off-diagonal, between unperturbed states are equal, given by the 4-point (zero range) instanton molecule interaction constructed by BLRS. For each channel, i.e., given spin and isospin, one collective state moves down to zero energy and all the other eigenstates remain at their unperturbed position of 1.5 GeV. Now the partition function is the sum of Boltzmann factors $Z(\tau) = \sum_n \exp(-\epsilon/\tau)$. For the moment we consider only momentum zero states. The zero-energy collective state will contribute unity to the sum. The other states will each contribute $\sim \exp(-1.5\text{GeV}/0.175\text{GeV})$ at $T = T_c = 175$ MeV, or $\sim 2 \times 10^{-4}$.

We thus approximate Z for a boson at rest, by

$$Z = \sum_n e^{-\epsilon_n/\tau} = 1, \tag{7}$$

keeping only the zero energy collective state. Thus, the free energy is

$$F = -T \ln Z = 0. \tag{8}$$

All that this shows is that in the construction of our chirally restored mesons, the large necessary binding energy reduces the large $\gtrsim 1$ GeV quark masses to zero, affecting neither pressure nor entropy, given by

$$p = -\frac{\partial F}{\partial V}, \quad \sigma = -\frac{\partial F}{\partial \tau}. \tag{9}$$

At temperatures well above T_c, where the vibrations are linear, and the epoxy is melted – roughly $T = 2T_c$ as will be discussed in the Appendix, we have, to

a good approximation, a Boltzmann gas, with each boson carrying $\epsilon \simeq 2.7T$ and $\sigma = 3.6$. The free energy is obviously a minimum.

However, the connection between equilibration and minimization of the free energy is not clear at T_c, and possibly not at $1.4T_c$, with the vicinity of which we are concerned. The usual derivation involves differentiating F to find the extremum

$$dF = dE - \tau d\sigma \qquad (10)$$

and showing that the right hand side is zero in thermal equilibrium. However, at T_c the bag constant B, which describes the energy in the epoxy, about half of the total bag constant (the soft glue being melted by T_c) increases the energy density without changing the entropy. It thus contributes negatively to the pressure. For many years lattice gauge simulations tended to find the pressure to be negative at T_c, which was, of course, unacceptable because the system would collapse. Brown et al.[19] in a crude calculation got the chiral restoration temperature to be $T_c = 172$ MeV just by requiring the restored phase to begin as soon as the pressure could be made positive.

Although we don't know whether the Boltzmann gas applies to the region around T_c where the vibrations are very nonlinear, we use it in Sec. 9 to discuss the density of the thermal excitations which must be added to bring the pressure to zero.

In the Harada and Yamawaki scheme[2] it is easy to see why the pressure in the hadron gas is low at T_c, because the interactions go to zero in the chiral limit. (Of course pressures must be equal in both phases in a phase transition.) In the resonance gas[20] the pressure on the hadron side is made small by putting most of the energy into the binding energy of excited states.

We can only conclude that the free energy probably is close to zero in the chirally restored phase at T_c, the value that results for heavy quarks from the Wilson loop.

7   The ρ-meson is special

As noted above, because of chiral restoration, we have only two types of chirally restored mesons π and ρ above T_c. The chirally restored A_1 is equivalent to the ρ and the chirally restored σ is equivalent to the π. Furthermore, because the $\vec{\tau}$ in the instanton molecule interaction is a four-vector, and all but the last term in the instanton molecule Lagrangian in BLRS[3], to which we return in discussion below, involve the square of $\vec{\tau}$, the different isospin states
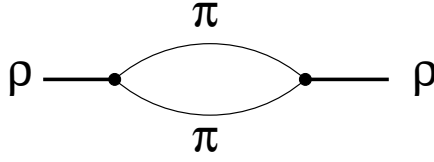
Fig. 7. Second order self energy contribution to the ρ mediated by 2π exchange.

are degenerate. In BLRS[3] we showed this to be nearly true for spin effects, which were negligible. Thus, we are left with only the ρ and π, with their various spins and isospins, in an SU(4) multiplet.

In the chiral limit the π is constrained as Goldstone boson to have zero mass at T_c (just below and just above). Therefore, only the mass of the ρ can be abjudicated; i.e., the only model dependence on meson masses at T_c comes in the ρ-mass. By T_c here we mean coming down to T_c from above; we know that going up to T_c below there is a fixed point in the ρ-mass of zero at T_c [2].

Although the mass of the ρ was found in classical approximation to be zero in BLRS[3], the instanton molecule fluctuated quantum mechanically about the time axis, with r.m.s. fluctuation in θ_4, the angle with this axis, of about 30°, so that m_ρ ended up at 560 MeV. However, our strong $\rho \leftrightarrows 2\pi$ coupling of Sec. 4 means that the lowest-lying eigenmode in the ρ-channel will be a "rhosobar"; in this case, a coherent linear combination of ρ and 2π, essentially

$$|\text{rhosobar}\rangle = a|\rho\rangle + \sqrt{1-a^2}|2\pi\rangle. \tag{11}$$

Consider the second-order self energy of the ρ from this coupling shown in Fig. 7 below. Of course this mixes the various 2π states only perturbatively with the ρ which is inadequate for a quantitative estimate.

If m_ρ is greater than $2m_\pi$, then this loop will involve a principal value integral with both positive and negative contributions. Thus, the greatest attraction is obtained when $m_\rho = 2m_\pi$. Thus, in the chiral limit $m_\rho \sim 0$, but with chiral symmetry breaking, $m_\rho \simeq 280$ MeV, twice the pion mass. We do not consider this a rigorous argument, but show below that the result is reasonable in the sense that going just below T_c where it is emitted the ρ gets its on-shell mass back. This procedure also minimizes the free energy (under the condition that the ρ has the chirally broken mass of $2m_\pi$) which should be achieved for equilibrium. We illustrate by this argument that chiral symmetry breaking will be very important in determining the mass of the ρ. Of course this sharp m_ρ is used only in lowest-order estimates. With the 380 MeV width we had in eq. (6) the ρ strength will be spread widely up to nearly 500 MeV.

Note that there is an equal degeneracy of longitudinal ρ's as pions at T_c. Therefore, if the ρ has a mass of ~ 280 MeV and we neglect the ~ 140

MeV difference of this mass from the pion mass, we would ultimately, as the temperature decreased towards thermal freezeout, get about as many pions from the decay of the longitudinal ρ's as from the pions themselves. This would be the situation if all interactions cut off below T_c, as envisioned by Harada and Yamawaki[2] where the width of the ρ goes to zero in the chiral limit. But, of course, this is not so good an approximation, especially not at T_c, where our estimates for $T_c + \epsilon$ are mixed with those of BSW[8] for $T_c - \epsilon$ through the explicit breaking of chiral invariance, which converts a phase transition into a smooth crossover transition.

In fact, Braun-Munzinger et al.[21] note a substantial discrepancy in the ρ_0/π^- ratio in semi-central Au-Au collisions. We give their discussion in what follows: These mesons have been reconstructed in STAR [22] via their decay channel into 2 charged pions. Comparing the preliminary results from STAR with the thermal model predictions of BRS[21], reveals that the measured values exceed the calculated values by about a factor of 2. This is unexpected, especially considering that BRS use a chemical freezeout temperature of 177 MeV for the calculation. One might expect these wide resonances to be formed near to thermal freezeout, i.e. at a temperature of about 120 MeV. At this temperature the equilibrium value for the ρ_0/π^- ratio is much smaller than the 0.11 found at 177 MeV. Even with a chemical potential for pions of close to the pion mass and taking into account the apparent (downwards) mass shift for the ρ_0 it seems difficult to explain the experimentally observed value of about 0.2. In fact, recently the STAR paper was published [23] where they give the final ρ_0/π^- ratio as $0.169 \pm 0.003(stat) \pm 0.037(syst)$ for peripheral Au+Au collisions, slightly lower than the preliminary ~ 0.2. Other statistical models [24,25] also have difficulty in explaining this large a ratio. We return to this below.

Now this excess is difficult to explain with the ρ width that of the free ρ, $\Gamma_\rho \sim 150$ MeV, during the drop in temperature from 175 to 120 MeV. This gives the strongest hadronic interaction and would surely be large enough to equilibrate the ρ's and π's down to thermal freezeout, the ρ's changing back and forth between two π's several times. Since they are obviously not equilibrated, the assumption that the ρ-meson has its free-particle width must be wrong, and we adduce the Harada and Yamawaki argument to say that only that part of the Γ which originates from explicit chiral symmetry breaking remains at T_c; in any case, that the in-medium width is substantially smaller than the free-particle one.

Let us try the extreme version of the theory in which the ρ and π mesons stop interacting as they drop below T_c in temperature. In Appendix C we develop a schematic model, which has the important ingredients, for how the ρ of mass $2m_\pi$ at T_c goes on shell before leaving the system. Out of the ρ_0's, only the longitudinal one will be of low energy, because of the instanton-molecule

polarization, as explained in BLRS[3]. Its strong attractive principal value interaction with the pion is maximized in magnitude by a mass of $m_\rho = 2m_\pi$ (so that there are no negative contributions to the principal value integral). We believe the nonzero longitudinal ρ mass to be the main effect of explicit chiral symmetry breaking. We use Boltzmann factors in our schematic estimates.

The total energy, rest mass plus thermal, of the 770 MeV ρ at $T_c = 175$ MeV is 1090 MeV, so the Boltzmann factor multiplied by 3 for the spins is 0.006, in the standard scenario. In our case only the longitudinal ρ has the low mass of 280 MeV at T_c. Its total energy is 652 MeV, with Boltzmann factor of 0.024. Thus, even though the degeneracy is cut down by 1/3, because of its lower mass, its abundance would be 4 times greater. Thereafter, in the chiral limit with $\Gamma_\rho = 0$, the ρ's would just leave. Given that the explicit chiral symmetry breaking is present and that the $T_c + \epsilon$ region will be overlapped with the $T_c - \epsilon$ region in a smooth crossover transition (see the next section) we would expect the abundance to be substantially decreased, but still present. The experimental enhancement by a factor ~ 2, half of our enhancement in the chiral limit, looks reasonable.

As the longitudinal ρ goes below T_c in temperature, it will gain back ~ 0.33 GeV binding energy it got from the color Coulomb interaction just above T_c, going on shell.[7] The reconstituted spin interaction also adds to the mass, $\sim 0.25(m_\rho - m_\pi) = 0.16$ GeV. The addition of these to the ρ-mass of 0.28 GeV at $T_c + \epsilon$ puts the ρ on shell at $T_c - \epsilon$.

Although our calculation is made only within SU(2), we believe that the spin effects will also be substantially weaker above T_c than below in SU(3). Now the $\bar{K}_0^\star$-K splitting below T_c is only about half of that of the ρ-π splitting, so the enhancement should not be so strong in the $\bar{K}_0^\star$ abundance as in that of the ρ, but a factor of ~ 2 in the $\bar{K}_0^\star$ abundance over that determined by equilibration at $T_c - \epsilon$ would only improve matters [1], although the difference between prediction and experiment is only one standard deviation in Ref.[1].

We did not find large deviations from the standard scenario in the STAR $K^{\star 0}/K$ and $\phi/K^{\star 0}$ ratios. We can see that medium effects will decrease both numerator and denominator here.

In our work we have relied upon chiral invariance to construct the π and σ so that their masses go smoothly through chiral restoration. We have not considered baryons and don't know how to introduce them. We do know that the Δ cannot have got much of its 300 MeV mass relative to the nucleon from the perturbative spin-spin interaction as in the MIT bag model. This would

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[7] We take the α_s below T_c to be that of charmonium, $\alpha_s \sim 0.33$. Just above T_c where the Coulomb binding is ~ 0.5 GeV (BLRS[3], Table 1), the effective α_s is $(\alpha_s)_{\text{eff}} \simeq 1$. So, the 0.33 GeV is 2/3 of the 0.5 GeV.

be absent about T_c, with the result that numbers of nucleons and Δ's would be equal there. Whereas STAR shows some excess of Δ's above the standard equilibrium scenario at central rapidity, this excess of $\sim 1/3$ is small compared with the $\exp(300\text{MeV}/T_c)$ that would be obtained in the perturbative scenario. More likely is that the singlet, isosinglet diquark which is strongly bound, and which exists in the nucleon, but not in the Δ, gives most of the energy difference between (possibly incipient) Δ's and nucleons above T_c. The binding of the diquark, as our Nambu-Jona Lasinio interaction, goes smoothly up through T_c since it results from the 't Hooft instanton interaction. Above T_c it will also have an attractive color singlet interaction. Thus, in some way the nucleon may be formed above T_c by the diquark picking up a quark, but the Δ would have to be formed by the coalescence of three quarks. In any case, the difference in energy between the incipient Δ's and nucleons just above T_c cannot be much smaller than just below T_C.

Of course, ratios of antiparticle to particle masses will be unchanged by whether equilibration is just below or just above T_c.

There are, thus, lots of ways in which the role of the medium dependence in equilibrium is hidden. It is no surprise that its role is clearest in the ρ/π ratio, because most of the ρ mass is generated dynamically in the chirally broken sector, whereas the pion, as a Goldstone boson, remains unaffected (except for the explicit chiral symmetry breaking). It is just this behavior of π and ρ which led to Brown-Rho scaling[27], which was first discovered empirically in nuclear spectra, because the π and ρ contribute to the tensor force with opposite signs. Thus, as the ρ-mass decreased with density, the ρ contributed more strongly and weakened the tensor force [28].

We remark briefly that our above discussion does not include the last term

$$\delta L_{\text{IML}} = -(\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \tag{12}$$

of the instanton molecule Lagrangian of eq. (37) of BLRS[3]. Above T_c with chiral restoration this is the same as

$$\delta L_{\text{IML}} = (\bar{\psi}\gamma_\mu\psi)^2; \tag{13}$$

i.e., attractive coupling of the ω-meson. This may be connected with a discontinuity in the baryon number chemical potential [29].

8 The Braun-Munzinger, Stachel and Wetterich (BSW) Scenario

Our scenario is that equilibration takes place just above T_c in the chirally broken sector. We have constructed a strongly interacting colorless liquid in that sector. Nonetheless, our scenario has important points in common with that of BSW [8], although they consider equilibration to take place in the chirally broken hadronic sector just below T_c.

These authors make 3 main points:

(1) Dominance of hadronic reactions with a high number of ingoing particles can be realized only very close to the phase transition.
(2) Two-particle processes are too slow to establish and sustain chemical equilibrium near the chemical freezeout temperature T_{ch}.
(3) Multi-particle scattering is indeed fast enough in order to maintain equilibrium for $T \geq T_{ch}$.

These authors carry out an extremely illuminating calculation of the rate r_Ω for Ω production, through the reaction $2\pi + 3K \rightarrow \bar{N}\Omega$. As we, they take perturbative (Boltzmann) thermal model densities. For the Golden rule matrix element they take the measured $p + \bar{p} \rightarrow 5\pi$ cross section. They check by evaluating the rate for Ω production[8] in a semi-classical approach, in which the standard two-body rate equation is generalized to multi-particle collisions, obtaining essentially the same result.

Thus BSW reach a time of 2.3 fm, sufficiently short to bring the Ω's into equilibrium. Close to T_c the Ω equilibration scales approximately as $r_\Omega \propto T^{-60}$!

Therefore, in a standard hadronic environment with reasonable parameters, chemical equilibration can be determined close to T_c, say at $T_c - \epsilon$.

In our paper above we have worked at $T_c + \epsilon$, i.e., in the chirally restored phase, from a different scenario, but with the same general results. Although as we formulated it, in the chiral limit, our scenario looks very different from that of BSW, we note that introducing the explicit chiral symmetry breaking would smooth our sharp chiral restoration transition into a smooth crossover one, mixing what happens in our scenario at $T_c + \epsilon$ to $T_c - \epsilon$. Also, even granted the Harada-Yamawaki results in the chiral limit[2], their ρ-meson width will not be zero with the explicit chiral symmetry breaking. Furthermore, with their T^{-60} factor for the Ω^-, equilibration would be likely to continue to $T_c - \epsilon$.

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[8] Because of the triple strangeness of the Ω, it may be considered the most difficult particle to equilibrate.
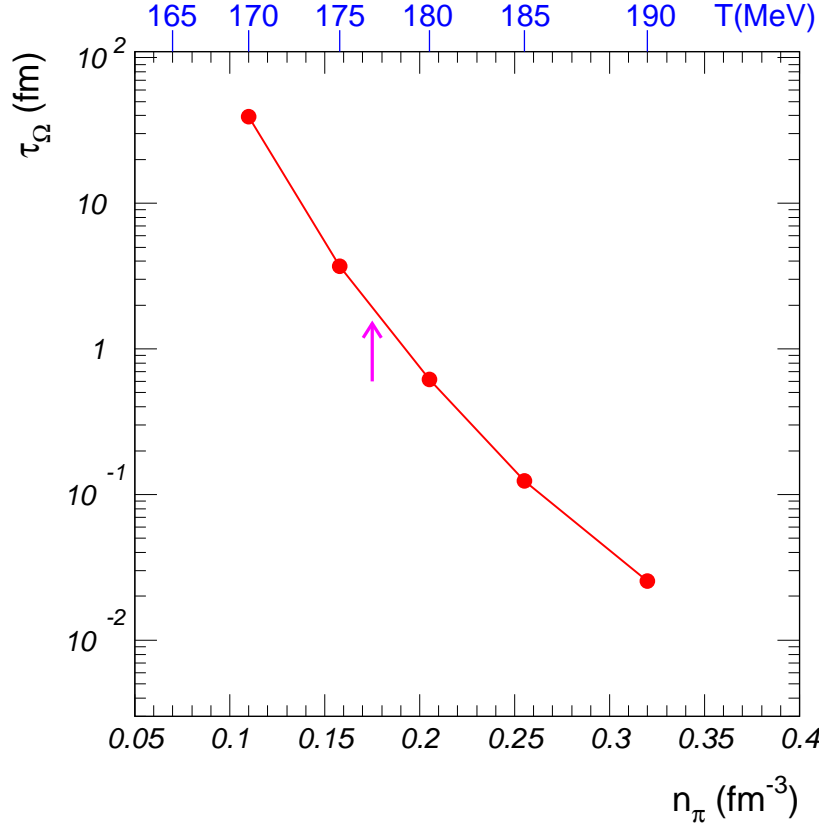
Fig. 8. Time $\tau_\Omega = n_\Omega/r_\Omega$ needed to bring Ω baryons into chemical equilibrium via multi-particle collisions. The arrow points to the Boltzmann pion density at T_c.

In practical terms, i.e., fitting the data, the BSW work has been extremely successful. Nonetheless, there may be exceptions such as we discuss with the ρ-meson, which cannot be explained in their work.

We note that inclusion of the $T_c + \epsilon$ region in their calculations can only help them. At $T_c + \epsilon$ we have 8 pions, as compared with 3 below T_c, the σ, ρ, isosinglet pseudoscalar and pions all being equivalent in the chirally restored region. The τ_Ω in Fig. 8 goes with the inverse 5th power of the pion density, so the higher degeneracy just above T_c would be of help.

Let us turn the previous statement around and say that the BSW work can help us. Namely, with the greater number of pions and the stronger interactions the material just above T_c is clearly well and truly equilibrated.

We believe that our scenario has the following merits:

(1) Chirally restored particles cannot leave the fireball. Therefore the latter

must cool until they can come down to T_c.

(2) However, equilibration at $T = T_c + \epsilon$ and the BSW equilibrium at $T = T_c - \epsilon$ which increases so strongly with T that even if zero in the chiral limit, is likely to be large because of explicit chiral symmetry breaking, will be mixed by the smooth crossover transition, over the region $\epsilon \sim 5$ MeV, the region below T_c in which BSW find equilibration. Thus, this BSW "band" will be the region in which thermally equilibrated mesons emerge.

We believe that the BSW paper and our note are the beginning of an unravelling of the puzzle of equilibration.

9   Discussion

Brown et al.[19] suggested that the mesonic resonance states π, σ, ρ, A_1 go smoothly through T_c with increasing temperature, simply changing from being chirally broken to chirally restored. The same general idea is retained in the resonance gas picture [20] in which resonances in addition to the above hadrons are included so as to make the transition smooth. As noted in Sec. 2, the π, σ, ρ, A_1 can be constructed as vibrations in the Nambu-Jona-Lasinio language below T_c so that our present scenario is not new in making them vibrations above T_c. However, it is easier to do the thermodynamics above T_c, because the $\bar{q}q$ molecules form a degenerate unperturbed configuration, so that all of the strength ends up in the collective state, none in the non-collective ones. This degeneracy endows the $\gtrsim T_c$ region with greatly enhanced interaction strengths.

The pressure must be nonnegative at T_c. This is achieved in the resonance gas picture by introducing mesonic excited states as well as resonances, as well as other hadrons than we consider.

Indeed, at T_c we will need additional degrees of freedom because in order to bring the ρ and A_1 masses to zero we must polarize them in the time direction, and this leaves us with only 8 degrees of freedom, which together with the π and σ give 16 at T_c. As noted, with this degeneracy we have only ~ 1 boson/fm^3 at T_c, which in Boltzmann approximation gives a pressure of $p = 0.9 T_c/\text{fm}^3 = 158 \text{MeV}/\text{fm}^3$, a bit higher when calculated in detail for bosons. Now in BGLR[14] we found that $\sim$ half of the ~ 500 MeV bag constant was melted in the soft glue by $T = T_c$, $\gtrsim$ half remaining in the epoxy, or hard glue. The -250 MeV/fm^3 contribution from this to the pressure leaves us with a deficit of ~ 100 MeV/fm^3.

A part of the pressure lost from the transverse ρ's and A_1's being high in mass

will be filled in by pressure from the glueballs. The soft glue that is melted as T goes up to T_c will go into glueballs, but these are high in mass below T_c, although certainly not higher than the $\sim 1.5 - 2.0$ GeV masses of glueballs at zero density. The Casimir operator is 3 for gluon pairs, compared with 4/3 for quarks, and so the color Coulomb should bring the gg bound states down by 1.25 GeV. Petreczky et al. find thermal gluon masses to be $\sim 20\%$ lower than thermal quark masses at $T = \frac{3}{2}T_c$ and $T = 3T_c$. With our assumed 1 GeV for the thermal quark mass, this would bring the gg bound states down to the region of masses where, with a multiplicity of 8 they could bring the pressure most of the way to zero.

Some pressure will come from the kaons: $K^+, K^-, K^0, \bar{K}^0$ and the vector[9] $K^\star(892)$ whose masses at the SU(2)×SU(2) $T_c \sim 175$ MeV should be somewhat decreased below the free masses by the melting of the non-strange quark condensates and partial melting of the strange quark condensates.

Hadrons other than our collective states will enter in and there will be some contribution from excited states of our chirally restored π, ρ, σ, A_1 although these states will lie at high energies because of the smallness of our molecules.

We see no problem in obtaining sufficient pressure well above T_c, say at $2T_c$, because the remainder of the bag constant will go as the epoxy is melted and our degrees of freedom will double to 32 as the ρ and A_1 are no longer polarized.

10   Conclusion

We have argued for equilibration of our excitations just above T_c. Every quark and antiquark within the relevant rapidity interval participate equally in a set of SU(4) vibrations, the energies of which go to zero in the chiral limit at T_c. At a higher temperature, approximately that reached in RHIC following the color glass stage ($t \sim \frac{2}{3}$ fm/c), those vibrations are found in the lattice gauge calculations of Asakawa et al.[6] and Wetzorke et al.[7].

The mesons remain equilibrated, at least in the chiral limit, as they are emitted, since the interactions have been found by Harada and Yamawaki[2] to go to zero as T goes up to T_c from below. And they cannot be emitted until T has come down to T_c from above. In the more realistic chirally broken system, we believe that chemically equilibrated mesons will emerge in the ~ 5 MeV band below T_c of Braun-Munzinger, Stachel and Wetterich [8].

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[9] In fact the vector masses may be considerably less, resembling to some extent the ρ-meson.

Whereas the zero masses of π and σ are protected by chiral symmetry at T_c, both slightly above and slightly below, the ρ-meson mass must go on shell at 770 MeV as it materializes, even though the abundance of ρ's is determined when its mass is nearly zero, slightly above T_c. In the chiral limit this means that the number of ρ's, which are reconstructed from their 2π decay, just below T_c should be greater than predicted by Boltzmann factor using their 770 MeV on-shell mass. A substantial excess in this number, which appears anomalous in the conventional treatment, should remain even after introduction of bare quark masses.

Interactions which dictate the above picture are strong with $g \gg 1$, very different from those in the perturbative quark-gluon plasma often predicted for RHIC energies.

We believe that we have established, backed by the LGS of Asakawa et al.[6], that the matter formed by RHIC is composed of giant collective vibrations of $\bar{q}q$ pairs and powered by the instanton molecule interaction. In a particle-wave duality that existed in the work of Nambu-Jona-Lasinio, these vibrations can also be interpreted as mesons. Above T_c, these mesons are chirally restored. In the formation of these colorless mesons, color is dynamically confined somewhat above T_c.

Normally one associates the large and rapid increase with temperature of the entropy s in the region of T_c, with the formation of the quark gluon plasma. However, Koch and Brown [30] (see especially Fig. 3) showed that hadrons going massless provided an increase in entropy that matched that found in LGS. In fact, somewhat above T_c (unquenched) where the ρ and A_1 are no longer polarized in the time direction, we have 32 degrees of freedom in the instanton molecule scenario. To the extent that these are essentially massless, these 32 bosonic degrees of freedom provide the pressure found in the SU(2) lattice gauge calculations [31]. Thus, one might say that the large increase in number of degrees of freedom results from the hadron masses going to zero in the chiral limit; i.e., Brown-Rho scaling [27].

Thus, RHIC has found a new kind of matter, one we find very interesting. In the sense that mesons are composed of quarks and gluons, it may be simple-mindedly called "quark-gluon plasma," but hardly so in the sense of the *predicted* perturbative QGP.

This same new form of matter must have been gone through in the early universe, as T decreased from $\sim 2T_c$ to T_c. The nice, smooth, essentially second-order transition we construct would mitigate against inhomogeneities originating in the chiral restoration transition. Since we have dynamical confinement just above T_c, we have not had to discuss deconfinement, which presumably requires a discussion of Polyakov lines.

Acknowledgments

A   Appendix A: Comparison with Lattice Gauge Simulation

As noted in BLRS[3] the main idea of Shuryak and Zahed [11] was that "after deconfinement and chiral symmetry restoration at T_c, nothing prevents the QCD coupling from running to larger values at lower momentum scale until it is stopped at the screening mass scale." The lattice scale is $\sim (0.5\,\mathrm{fm})^{-1} \sim 400$ MeV because for higher scales (shorter distances) the charges are locked into the quarks and antiquarks. In order to compare with the lattice results on α_s shown in Fig. 2, we use the perturbative (long distance) scaling

$$\frac{\alpha_s(T \gtrsim T_c)}{\alpha_s(T < T_c)} \simeq \frac{\ln(\Lambda\chi/\Lambda_{\mathrm{QCD}})}{\ln(400\,\mathrm{MeV}/\Lambda_{\mathrm{QCD}})} \simeq 3 \tag{A.1}$$

where we have taken the chiral symmetry scale $\Lambda_\chi = 1$ GeV and $\Lambda_{\mathrm{QCD}} = 250$ MeV. The actual increase in Fig. 2 from the $\alpha_s \sim 1/3$ for isolated charmonium to $\sim 8/3$ at T_c, where we have included the Casimir operator of $4/3$, is more than double this, but at least of the right general size, leaving little doubt that the increase in color Coulomb at T_c results from movement towards the infrared with chiral symmetry restoration. We show below that the instanton molecule interaction lowers the scale Λ essentially to zero. BLRS[3] found that the heavy quark Coulomb interaction provided only $\sim 1/8$ of the attractive interaction which brought the π and σ masses to zero at T_c, so the movement towards the infrared is much more pronounced than given by the lattice results, which do not include the velocity-velocity interaction and instanton molecule interactions, neither of which would contribute appreciably to the heavy quark situation.

The above discussion refers to the large distance behavior. The shorter dis-

tance interaction is strongly influenced by the necessity of the interaction going to zero as $r \to 0$, because of asymptotic freedom.

Preliminary results on quenched QCD [13] and in full QCD [32] are given in [33]. The chief result of full QCD was to rescale by the T_c (unquenched); i.e., results at $T_c(quenched) \sim 260$ MeV were moved down to $T_c(unquenched) \sim 175$ MeV. Whereas we believe this to be true for the heavy-quark color Coulomb (singlet) interaction, we also believe that for other problems the instanton molecule interaction may introduce an additional scale, as we discuss below.

The lattice calculations show a great dependence upon radial coordinate r, as might be expected from asymptotic freedom and confinement. We showed the α_s from the Polyakov loops in Fig. 2, where α_s is extremely large at T_c.

However, when all is said and done, the singlet potential energy just above T_c can be schematized

$$V = \frac{\alpha_s}{r} \quad (r > \hbar/2m_q) \tag{A.2}$$

with $\alpha_s = 0.5$ to the accuracy at which we can read off the LGS curves. This potential was the one used in BLRS, the Coulomb interaction being modified for $r < \hbar/2m_q$, which can be thought of as imposing asymptotic freedom. Furthermore, this schematic potential should work reasonably well up to $T \sim 1.4T_c$, where the screening mass scale can be read off as ~ 0.5 fm, distances below this scale being important in the region of $T_c \sim 1.4T_c$. We believe that the above schematization is adequate to discuss physical phenomena in this range of energies. From $T \sim 1.4 - 1.5T_c$ the Debye screening sets in, so that the instanton molecules become bigger and by $T \sim 1.9 - 2T_c$ they become unbound. We leave this latter region to Shuryak and Zahed[11].

Whereas we must increase the heavy-quark Coulomb interaction for $T \gtrsim T_c$ by a factor of ~ 8 in order to include the light quark effects, we have only the underpinning of the heavy quark LGS, aside from our general argument that $m_\pi = m_\sigma = 0$ at $T = T_c$ in the chiral limit. The LGS show, however, that the screening distance of ~ 0.5 fm is not reached until $T \sim 1.4T_c$, so that the charges are still locked into the quark and antiquark up to this temperature to a good approximation. This was all that was needed to construct the instanton molecule interaction in BLRS, so we assume that there is little change in the interactions up to this temperature, although Debye screening sets in rapidly above.

In fact our argument that our scenario at T_c should hold well up to $1.4T_c$ is improved if we construct the collective wave functions. In BLRS we calculated the instanton molecule wave functions with only the color Coulomb and velocity-velocity potential; i.e., with an $\alpha_{s,\text{eff}} = 1$. This gave us an rms radius
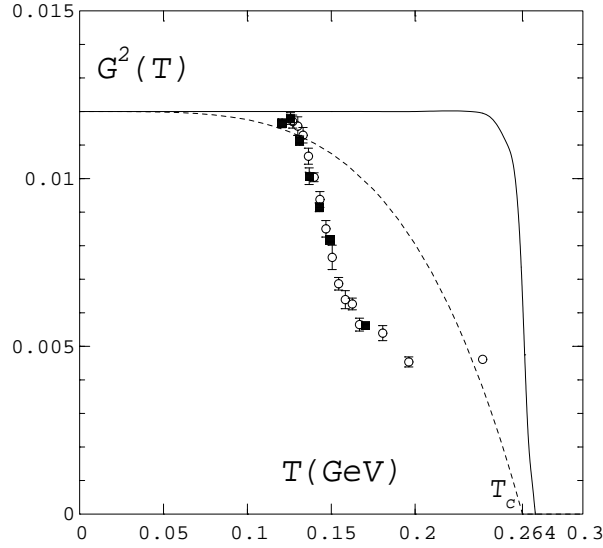
Fig. A.1. Gluon condensates taken from Miller (Fig. 2 of BGLR[14]). The lines show the trace anomaly for SU(3) denoted by the open circles and the heavier ones by filled circles. The T_c marked in the figure is that for quenched QCD, whereas we deal with unquenched QCD with $T_c = 175$ MeV.

for the π and σ of $\sim 1/3$ fm. However, the energy of the π and σ were lowered only $\sim 1/4$ of the way from 2 GeV to 1.5 GeV, as seen in Table 1 of BLRS. We then calculated that the instanton molecule interaction would further lower the energy to zero, but we did not calculate the wave function of the collective state.

This wave function can be expressed in relative momenta q of quark and antiquark. If the energy is to be lowered by 2 GeV, then this relative momenta must be made up of components that extend to relative momenta of this size; i.e., the coherent wave function, solution of the $\bar{q}q$-bubble sum, must be of radius $\sim \hbar/2m_q c$, or ~ 0.1 fm. In other words, the collective wave function made up of all quark-antiquark, or quark, quark-hole components must be really tiny if the collective state it is to describe, is to be bound by $2m_q c^2$. Thus, the electric charge is really locked into a tiny volume, much less at T_c than the screening radius. This is why we believe that there is little change in the π and σ masses between T_c and $1.4T_c$.

In fact, we see some indication of the constancy in gluon condensate between $T_c = 175$ MeV and $1.4T_c$ from the LGS of Miller[34] which we reproduce as our Fig. A.1. The initial drop in gluon condensate (trace anomaly) up to $T_c \simeq 175$ MeV is shown in BGLR[14] to be given by the melting of the soft glue, the glue connected with chiral symmetry restoration. It is connected quantitatively in BGLR with the energy provided by the dynamical (or constituent) quark masses going to zero. The last two points on the right show that the "epoxy", the condensate of hard glue connected with instanton molecules, does not

25

change perceptively from $T_c = 175$ MeV to $1.4T_c$. This is just the region we discussed above in which the electric charges are trapped in the small instanton molecules inside of the screening radii. The large NJL forces, which bring the molecule masses to zero or nearly to zero in the region T_c to $1.4T_c$ clamp the hard glue tightly in place.

We also got a simple estimate of the molecular breakup temperature from our argument that quark and antiquark momenta q must be $\sim m_q c$. There will be break up once the thermal energy of q is equal to $m_q c^2$. But relativistically $q \sim 3T$. Thus $T_{zb} \sim m_q c^2 / 3$, roughly $1.9T_c$ (unquenched).

We now repeat here our treatment of the instanton molecules and their connection with the giant resonances found in the lattice calculation of Asakawa et al.[6] since Ref.[5] is out of print. We have established that our $\bar{q}q$ (or quark, quark-hole) representation is that of the Coulomb states. Every $\bar{q}q$ state is connected to every other $\bar{q}q$ state by the 4-point instanton molecule interaction as in Fig. A.2.

We construct this interaction either for quark-particle, quark-hole states or for quark-antiquark states, as noted in BLRS. Initially for simplicity we consider only states of total momentum zero; i.e., the $\bar{q}q$ state has no translational energy. The latter will be put in later.

As in BLRS we take the quark and antiquark masses to be 1 GeV, considering all $\bar{q}q$ state energies initially at 2 GeV to be lowered to 1.5 GeV by the color Coulomb interaction. The diagonalized collective state will turn out to be a boson state at zero energy and zero momentum. We will then treat this state as a boson, neglecting the incoherent states which remain at 1.5 GeV, where their Boltzmann factors at T_c are so small that they can be neglected.

The instanton molecule interaction starts out as a nonlocal one, the nonlocality being governed by the $\bar{\psi}\psi$ of the instanton zero modes. The $\bar{\psi}\psi$ is sharply peaked, mostly lying within a radius of $r \sim \sqrt{2/5}\rho$, where $\rho \sim 1/3$ fm is the radius of the instanton (Appendix of BLRS[3]). This regulates the nonlocality and acts as a cutoff in the possible momentum range of the $\bar{q}q$ states. The way this is handled in BLRS is to use a δ-function 4-point interaction, but to cut it down by a factor $F = (0.75)^2$. The underlying nonlocality restricts us to a band in momentum space.

We have, therefore, several approximations, but we are guided in our numerics by the principle that the pion energy must turn out to be zero in the chiral limit in order to make a smooth chiral restoration transition. Of course, the chirally restored σ-meson must also have zero mass at $T = T_c$.

If we keep only $\bar{q}q$ states going forward in time (ignore ground-state correlations) we have the quantum mechanical problem to solve, for $T = T_c$,
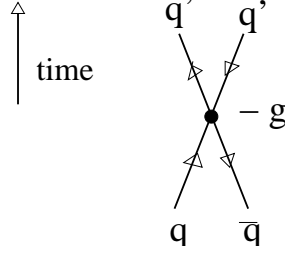
Fig. A.2. Instanton molecule interaction g which connects $\bar{q}q$ bound states.

$$\left[-0.5 \text{ GeV } \mathrm{I}_{N \times N} - g_I \begin{pmatrix} 1\ 1\ \cdots\ 1 \\ 1\ 1\ \cdots\ 1 \\ \cdot\ \cdot\ \cdots\ \cdot \\ 1\ 1\ \cdots\ 1 \end{pmatrix}_{N \times N} \right] \psi_N = E\psi_N. \tag{A.3}$$

In other words, having reduced the instanton molecule interaction to a 4-point one, all matrix elements in the secular matrix are equal. Here g_I is the instanton molecule interaction. The first -0.5 GeV times the N-dimensional unit matrix $\mathrm{I}_{N \times N}$ is just the color Coulomb interaction.

Now from the principle of insufficient reason (all off-diagonal matrix elements are equal) we know that one eigenfunction (the collective one) is

$$\psi_{\text{coll}} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{pmatrix}_N \tag{A.4}$$

and we easily find that

$$E = -0.5 \text{ GeV} - N\ g_I. \tag{A.5}$$

In BLRS[3] we accomplished the same diagonalization by a sum of $\bar{q}q$ loops going forward in time, giving

$$Ng_I = 0.75 \text{ GeV}. \tag{A.6}$$

Thus, at this level the collective state is brought down from the $2m_q = 2$ GeV to 0.75 MeV.

The equivalence between solving the secular matrix and summing loops as was done in BLRS going forward in time was developed in detail by Brown
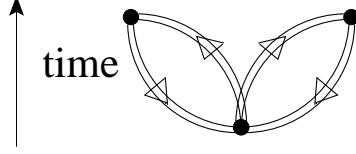
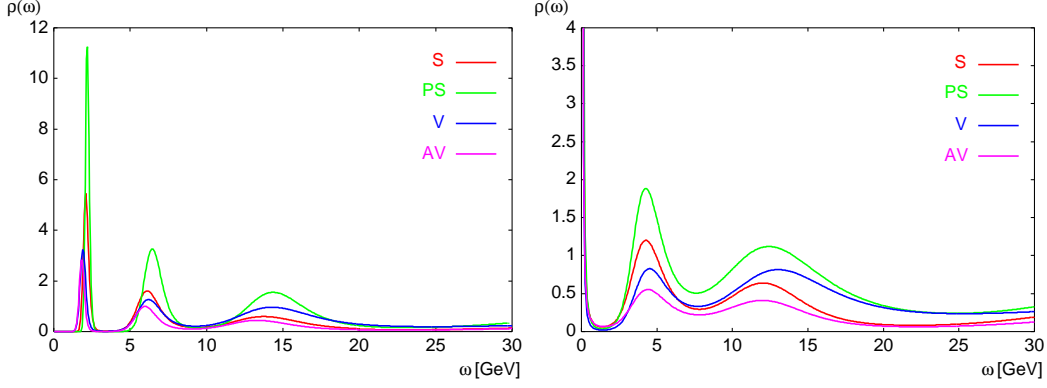Fig. A.3. Backward-going loops; equivalently, ground-state correlations.

Fig. A.4. Spectral functions of Asakawa et al.[6]. Left panel: for $N_\tau = 54$ ($T \simeq 1.4T_c$). Right panel: for $N_\tau = 40$ ($T \simeq 1.9T_c$).

[5]. There it was shown that if the collective state is brought down half-way to zero (In our above case from 1.5 GeV to 0.75 GeV by forward going loops.) then with inclusion of backward going ones it will be brought to $E = 0$. The backward-going diagrams are really ground-state correlations, are shown in Fig. A.3.

We consider only the fundamental mode of the vibrations, the higher modes being lattice artifacts [35,36].

Let us consider the left panel of Fig. A.4, taking the fundamental to be at 2.1 GeV. As noted, BLRS[3] had no isospin dependence and very small spin dependence, so the SU(4) character of the vibrations is explainable. Now we developed above that the situation at $T = 1.4T_c$ should not have changed much since T_c, because the screening scale has only reached that of the instanton molecules about there.

In fact, the calculation in BLRS[3] used the Coulomb wave functions and then summed the loops with four-point interaction given by the instanton molecule one. The very small collective wave function of size $r \sim \hbar/2m_q$ was not constructed. But the Coulomb potential was chosen so that $dV/dr|_{r=0} = 0$ and the instanton molecule source $\bar{\psi}\psi \sim (r^2 + \rho^2)^{5/2}$ also had $dV/dr = 0$, both of which should follow from asymptotic freedom. Furthermore, since the molecule energy is a minimum at T_c, it should recover (from zero) only quadratically as $(T - T_c)^2$ from its minimum. All of these effects make the growth in molecule slow as T moves above T_c, so that situation at $T = 1.4T_c$ should not be much different from $T = 0$.

Neglecting small changes from T_c to $1.4T_c$ in m_σ and m_π, we find that with $m_q = 1.2$ GeV, giving unperturbed molecule energy of $m_q + m_{\bar{q}} = 2.4$ GeV, then the heavy quark Coulomb potential will bring this down $\sim 1/8$ of the way to zero, or to 2.1 GeV. This m_q is $\sim 20\%$ below the Petreczky et al. 1.5 GeV mass from quenched calculation at $3/2\ T_c$, but still above the 1 GeV that would be applicable if m_q scaled with T_c in going from quenched to unquenched calculations. Although this difference is probably not larger than uncertainties in the LGS, we believe that it is indicative of another scale than the color Coulomb one, that of the instanton molecules, between T_c and $1.4T_c$.

In any case, the fact that the LGS find a vibration of energy $\hbar\omega \sim 2$ GeV at $T = 1.4T_c$ and that the vibration requires attraction between quark and antiquark, shows that $m_q > 1$ GeV.

Now using the fact that the LGS have only $\sim 1/8$ of the total attraction, we find that the m_σ and m_π would still be essentially zero at $T = 1.4T_c$. We confess that our above scenario seems to fit a complicated situation too well, but believe it to be roughly true. Notice that at $T = 1.4T_c$, where we have argued the situation is not much different from T_c because the charges are locked into molecules inside of the Debye screening, the LGS show a complete SU(4) symmetry. Harada and Yamawaki[2] show that in the chiral limit the ρ mass joins the π and σ mass as T goes upwards to T_c at zero. It seems most reasonable that the ρ mass is also zero just above T_c.

Once the screening range of ~ 0.5 fm is reached by the size of the instanton molecules, the latter would be expected to become rapidly unimportant, probably by $T = 1.9T_c$ where the situation appears to have gone perturbative [3] or at least to have reached the breakup of the $\bar{q}q$ bound states. Since the instanton molecules are unimportant at $1.9T_c$, that temperature should scale with T_c in going from quenched to unquenched; thus in full QCD it should be $1.9T_c(unquenched) \simeq 332$ GeV, just above the temperature at RHIC following the color glass stage.

In fact we referred to the LGS as having seen vibrations. From our above discussion it should be clear that they saw only the $\bar{q}q$ bound states, and only the binding of these by the Coulomb interaction, not including the velocity-velocity interaction or instanton molecule interaction which do not contribute for heavy quarks. The real vibrations are those we calculated earlier in this section, with inclusion of backward going graphs (ground-state correlations). In order to get the full collectivity these latter effects have to be included, but they cannot be in Euclidean time LGS. However, they can be added theoretically as we did for $T = T_c$.

Thus, for the modes such as the π and σ which go massless at T_c, the best that even an unquenched calculation (full QCD for light quarks) can do is to bring

them down halfway to zero because the ground state correlations cannot be handled in Euclidean space. (An exponentially decreasing antiquark cannot be a quark going backward in time.) (We noted above that in Minkowski space, if the forward going graphs brought the vibrations down halfway, then the inclusion of backward-going graphs would bring them down all the way. Note that our schematic mode eqs. (A.3)-(A.5) is equivalent to a treatment with only forward-going graphs.) In this case, because the diagonalization of the secular matrix puts all of the trace into the one collective state for each spin and isospin, the strength in this one collective state is equal to the sum of the strengths in all of the individual $\bar{q}q$-molecules, although the energy of the collective state will be moved down 4-times further at T_c.

The theory of vibrations has many names; e.g., random phase approximation, particle-hole solutions of the Bethe-Salpeter equation, bosonization of particle-hole excitations, time-dependent Hartree-Fock, linearization of the equation of motion. The latter carries the best description; they are the best that can be made from linearization. They are also the lowest-order approximation that satisfies conservation equations; e.g., conserves Newton's law[37].

It may seem somewhat amazing that we can apply the same theory of vibrations in matter at temperatures of ~ 200 MeV as we do in nuclei at low energies. However by eliminating longitudinal and scalar gluons in favor of an instantaneous Coulomb interaction [10] which gives the Furry interaction, and then, integrating over time and regularizing by integrating over the space-like nonlocality, one gets an $\sim 50\%$ reduced, but instantaneous interaction (Appendix of BLRS [3]). Thus, we have the same tools in hand as in low-energy nuclear physics with the added simplicity that our Furry representation quark-antiquark states are really degenerate, as confirmed by the LGS. (In low-energy nuclear physics the term energy in the giant dipole excitation $\hbar\omega$ is the distance between shells but the unperturbed energies are split by the spin-orbit interaction which gives added structure[17].) Thus, we do have the same problem, but heavy quark LGS have seen only a small part of it, as we laid out above. Nonetheless, the small parts of the "iceberg" seen in the LGS give already the most prominent excitations above T_c.

There may, in fact, be an advantage in only a small part of the light-quark interaction entering into the LGS. The vibrations look pretty linear at $T = 1.4T_c$. However, with the ~ 8 times larger interaction and the strong $\rho \leftrightarrows 2\pi$ mixing we have investigated, the widths of the ρ and π might have been spread out over such a wide range of energies that the vibrations would not be discernible from the background.

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[10] Note the movement towards the strong $N = 4$ supersymmetric Yang-Mills theory at finite temperature in which the scalar and longitudinal phases travel faster at superluminal speeds as the coupling becomes stronger [38].

B Appendix B: Consequences of Rescaling

We needed Appendix A in order to discuss the physics accompanying rescaling, primarily to clarify that the LGS takes note of the change in scale from $\Lambda_\chi = 1$ GeV to the instanton molecule (or, equivalently, chirally restored meson) scale set by their zero masses, in the chiral limit. Thus, although we go up in temperature to T_c, suddenly the scale Λ_χ governing the behavior of our thermodynamic quantities drops out, and the system reverts way back toward the infrared. However, until chiral symmetry is restored, the system knows nothing about the infrared behavior that enters at T_c.

Thus, within the chirally broken system there is a movement towards weak coupling in the effective variables, i.e., the hadrons, resembling asymptotic freedom. At the risk of a misnomer (since non-asymptotic fixed point is involved), we shall call this "effective-sector asymptotic freedom." Beta functions are written down in these variables, the vector mesons replacing gluons, etc. in the hidden local symmetry of Harada and Yamawaki[2]. The flows of various quantities in their renormalization group treatment join smoothly with those of QCD, so their theory in the effective sector mimics QCD, even to the point of having a local gauge symmetry: The critical point T_c delineates flavor gauge theory from color gauge theory, the latter ceding to the former in a smooth way [11]. One may say that the HLS theory is the shadow of QCD in the effective sector. It even has its own gluon condensate [14], the soft glue which melts as the temperature increases towards T_c (see fig. A.1) although the effective sector knows nothing of the hard glue, the explicitly broken chiral symmetry above T_c. Thus, in their own effective world the effective theory has a gauge theory description, effective asymptotic freedom, etc. It gets its scale from the spontaneous breaking of chiral symmetry through the condensate of soft glue [27] as shown in Fig. A.1.

In fact, from the region in which $T \sim 125$ MeV up to $T_c = 175$ MeV the system behaves as if it is in a mixed phase with nearly constant pressure since as described in [14] the increase in energy with temperature goes into melting the soft glue. Thus the elliptic flow is very low and, indeed, collapses around mid-rapidity for protons in the NA49 40 AGeV experiments [39]; the velocity of sound $v = p/E$ decreases with increasing energy. It is in this region that the Harada and Yamawaki[2] scenario of vector manifestation brings out the effective sector asymptotic freedom. The hadrons are mainly pions and vector mesons and the fixed point of $m_\rho = 0$ and $g_V = 0$ as T_c is reached from below comes in to further soften the interactions. In BGLR [14] we showed that the nucleons dissolved into constituent quarks as the soft glue is melted, the

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[11] This "continuity" of gauge degrees of freedom between color gauge and flavor gauge was already conjectured in [40].

quarks then going massless, in the chiral limit, as T moves up to T_c. As noted at several places in our paper, the pressure is approximately, possibly exactly, zero at T_c. (The free energy F is essentially $-pV$ for our purpose, so making it zero makes the pressure zero.) All of this is efficiently encoded in the HLS with the vector manifestation, HLS/VM, i.e., the effective gauge theory in effective variables. Whereas it is true that this theory thus far describes only the behavior of pions and vector mesons, and has not been extended to include baryons [12], the former are the important variables for the thermodynamics of the relativistic heavy ion collisions.

Now the effective sector given by the Harada-Yamawaki theory which is not valid above T_c knows nothing about the sector in which chiral symmetry is restored. Before Harada and Yamawaki RG[2], it was clear that something must change, because the Hagedorn temperature limited the increase in T_c. What we have shown is that simply to enforce the most basic (and simple) requirement that the Goldstone boson, the pion, move up smoothly through T_c without changing mass, requires the weakly interacting effective sector at $T_c - \epsilon$ to be replaced by the most strongly interacting sector in the gamut of the system, and this is what RHIC has done.

The "new" system, representing bona-fide QCD, has been greatly studied and described. It has its own gluon condensate, the hard glue or epoxy, which explicitly breaks chiral symmetry. QCD has its own running coupling constants, so the reincarnation of asymptotic freedom, the shadow of which occurred just below T_c in the behavior of the effective variables, occurs only higher up, at $T \gtrsim 2T_c$, possibly above the maximum temperature reached at RHIC.

We thus see that our rescaling arguments reverse the generally accepted scenario. Going up in temperature we first encounter a weakly interacting region in the effective variables, just below T_c. With chiral restoration this gives way to the most strongly interacting matter [13] encountered in heavy ion collisions.

With the reversal of the commonly accepted scenario, the weakly interacting system below T_c changing into the strongly interacting system just above T_c, it is clear that equilibrium has to take place in the latter region. We have adduced arguments from LGS and from experiment (especially the ρ-meson decay) to support this new and surprising scenario.

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[12] It has been shown however that the VM holds also for the constituent quarks, so the dynamical quark mass vanishes at the chiral transition [41].

[13] Recall that the pion mass is brought down from $m_q + m_{\bar{q}} \sim 2$ GeV to zero by the interactions!

C Appendix C: A Schematic Model for How the ρ Goes On Shell

Shuryak and Brown [42] calculated that the ρ-meson freezes out at a temperature of 120 MeV and a baryon density of 15% nuclear matter density. The ρ mass at this density was measured by STAR [22] to be 700 MeV. The ρ then decoupled from the system and got the remaining $\sim 10\%$ of its on-shell mass back from its kinetic energy. In the regime below 125 MeV density dependent effects, which we do not discuss here, dominate, but from 125 MeV up to $T_c = 175$ MeV thermal effects connected with the melting of the soft glue predominate[14]. In this paper the constituent quark mass was correlated with the binding energy of the soft glue (which is responsible for the dynamical breaking of scale invariance; i.e. Brown-Rho scaling. The soft glue was shown in Fig. A.1. Shuryak and Brown (see Sec. II.E of Ref. [42]) argued that collision broadening built the $\Gamma_\rho^\star$ back up to the on-shell $\Gamma_\rho = 150$ MeV at $T_{\text{freeze out}} = 120$ MeV; the part of the width connected with the two-π decay being brought down to 100 MeV because of the reduced (p-wave) penetrability involved in the decay. We consider here only the ρ connection with the two-π system.

In the Harada and Yamawaki[2] scenario $m_\rho^\star$ and $\Gamma_\rho^\star$ go to zero in the chiral limit as T goes to T_c from below; also $g_V^\star/m_\rho^\star$ goes as a constant as T goes to T_c. This means that [14]

$$\frac{\Gamma_\rho^\star}{\Gamma} \longrightarrow \left(\frac{m_\rho^\star}{m_\rho}\right)^3 \left(\frac{g_V^\star}{g_V}\right)^2 , \tag{C.1}$$

three powers coming from the p-wave penetrability and two from $(g_V^\star)^2$.

We now make the assumption that the dynamically generated part of the ρ-mass scales with the constituent quark mass in the chiral limit. This is a reasonable assumption near T_c as it was near n_c [41], but we extrapolate it all

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[14] Note that from Sec. II.E of Shuryak and Brown[42], $\Gamma_\rho^\star/\Gamma_\rho$ would go to zero as $(m_\rho^\star/m_\rho)^3$ even if $g_V^\star$ did not scale, because of the p-wave pion penetrabilities and reduced phase space for decay. Indeed the expression for the decay of the free ρ is

$$\Gamma_\rho = \left(\frac{g^2}{4\pi}\right) \frac{m_\rho}{12} \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{1/2} .$$

If we replace m_ρ by $m_\rho^\star$ and let the 700 MeV nearly on-shell mass found at thermal freezeout by STAR[22] go to zero, this expression gives a $\Gamma_\rho^\star$ that scales rapidly dropping to 44 MeV by the time $m_\rho^\star = 350$ MeV. Thus, our decoupling of the ρ from the pions does not depend much upon the scaling of $g_V^\star$; matters would not be much changed if one had only BR scaling.

T [MeV]	$\Gamma^\star$ [MeV]	$(\Gamma^\star)^{-1}$ [fm]
125	100	2.0
135	53	3.8
145	25	7.9
155	11	18
165	4	53

Table C.1
Effective decay widths and mean free paths for various temperatures.

of the way until the constituent quark gets nearly all of its mass back. From BGLR[14] we have the binding energy of the soft glue going as

$$\text{B.E.} = 12 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \left(\sqrt{k^2 + m_Q^{\star 2}} - |k| \right). \tag{C.2}$$

We recall that this equation is motivated by Nambu-Jona-Lasinio in which the binding energy in the vacuum – in this case that of the soft glue – is obtained in terms of the negative energy quarks in the condensate which breaks chiral symmetry having dynamically generated masses $m_Q^\star$.

For $m_Q^\star \ll k$ we get from (C.2)

$$\text{B.E.} \approx \frac{3}{\pi^2} \frac{\Lambda^2}{2} m_Q^{\star 2}. \tag{C.3}$$

We then make the simplest schematic model incorporating the above ideas as

$$\Gamma_\rho^\star \approx 100 \text{ MeV} \left[\left(1 - \frac{2m_\pi}{700 \text{ MeV}} \right) \left(\frac{T_c - T}{50 \text{ MeV}} \right) + \frac{2m_\pi}{700 \text{ MeV}} \right]^5 \tag{C.4}$$

which comes to 100 MeV at T=125 MeV rather than at 120 MeV as found by Shuryak and Brown[42], but we want a point midway between 120 and 130 MeV to do our later integration as the STAR experiment[22] does not constrain Γ accurately. (In fact, there are indications that at densities $< n_0$ and probably also at low temperatures – see Fig. 2 of Brown and Rho [43] – $g_\rho^\star$ does not scale as rapidly as $m_\rho^\star$. This would move the point where $\Gamma_\rho^\star = 100$ MeV lower.) We then find the results in Table C.1, where $(\Gamma^\star)^{-1}$ gives an indication of the mean free path in fermis.

As noted in BGLR[14], this region of temperatures is in the environment usually described as a mixed phase. Rather than being at constant temperature

34

$T_c = 175$ MeV, where the bag constant is usually assumed to be a source of energy going down in temperature, so that it keeps the temperature constant in a real mixed phase, in our case (as shown in the lattice calculations, Fig. A.1) the energy is furnished over a relatively large (175 - 125 MeV) range of temperatures, giving an ~ 50 MeV range of mixed phase. The usually assumed ~ 5 fm/c from phase transition to freeze out, means that each interval of 10 MeV takes the system ~ 1 fm/c to traverse. We see that the ρ mesons are never really equilibrated, although their final decay rate just before freezeout would be sufficient to equilibrate them had it acted for the full ~ 5 fm/c. They mostly decay in the last 2 fm/c, centered about $T = 130$ MeV; i.e., from 140 to 120 MeV, the latter the freezeout temperature. In that interval, by taking the average mean free path as 2.5 fm, roughly $\exp(-2\text{fm}/2.5\text{fm}) = 0.45$ of the ρ mesons don't decay into two pions. Now the pions are always in equilibrium (with themselves). Thus, the rate at which pions create ρ mesons in the last 2 fm/c would be equal to the rate at which ρ mesons go into pions, were there equilibrium. But the pion equilibration doesn't depend appreciably on the ρ mesons, so we can use the equilibrium $\rho_0/\pi^- = 0.08$ [24]; i.e., $(1 - \exp(-2/2.5)) \times (\rho_0/\pi^-) = 0.55 \times 0.08$ will be created from the pions in the last 2 fm/c. (This reverse process uses the same $\Gamma_\rho^\star$ as the ρ-decay.) Thus, at thermal freezeout we have $0.45(\rho_0/\pi^-)_{\text{initial}}$ left, plus the $0.55 \times 0.08(\rho_0/\pi^-)$ produced in the last 2 fm/c from the pions, the sum of which we equate with the STAR result

$$0.45 \times (\rho_0/\pi^-)_{\text{initial}} + 0.55 \times 0.08 = 0.169 \pm 0.037 \qquad (\text{C.5})$$

which gives

$$(\rho_0/\pi^-)_{\text{initial}} = 0.20 - 0.36. \qquad (\text{C.6})$$

The systematic error of ± 0.037 gives the rather large uncertainty in the initial number of ρ's. We come to the factor of $\sim 2 - 4$ increase over the Braum-Munzinger et al's 0.11 value of (ρ_0/π^-) equilibrated at 177 MeV assumed just below T_c. In the text we used $m_\rho^\star = 2m_\pi$ just above T_c so as to minimize the free energy, but with out estimated $\Gamma_\rho^\star \sim 380$ MeV just above T_c the $m_\rho^\star$ will be spread over a broad peak lying somewhat below the on-shell ρ mass of 770 MeV.

We believe our schematic model illustrates the main physics:

(i) The explicit chiral symmetry breaking is important, giving the $2m_\pi$ in the ρ-mass which roughly gives the center of the ρ mass distribution.

(ii) The interaction of the ρ with the pions is negligible at T_c, increasing as T drops, and becoming large at the end of the mixed phase as the ρ is nearly on shell.
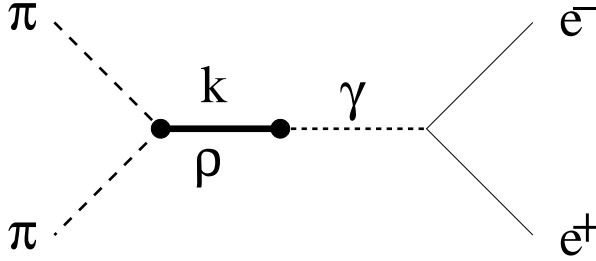
Fig. C.1. Dilepton production through the ρ.

(iii) The initial abundance of ρ's is large, because they are equilibrated just above T_c with a spread of masses well below the mass of the on-shell ρ.

Given (iii) it is very reasonable that some of this initial abundance remains at thermal freezeout. In fact, we believe in the lower limit of at least twice that of the standard scenario in which the ρ freezes out just below T_c, because with large pion and ρ chemical potentials it seems possible to nearly preserve the factor 2 from chemical to thermal freezeout.

As suggested before, CERN, in finding a weakly interacting system, was operating mainly in the mixed phase described in Appendix B. RHIC clearly comes down through the region of temperatures well above T_c, through $1.4T_c$ down to T_c where we have our low-mass chirally restored mesons. In this ~ 2 fm/c the ρ-mesons have time to decay into dileptons. This should substantially increase the invariant masses in our range up to ~ 400 MeV and possibly higher.

We now go on to discuss how our scenario of equilibrium above T_c will affect the dileptons.

Vector dominance is known to be violated due to the vector manifestation fixed point $a = 1$ at finite density and/or temperature [2]. However the "intrinsic" violation more prominent in the baryonic sector [9] is not expected to make a qualitative influence in the mesonic sector, so we will simply adopt the vector dominance in our discussions. We need however take into account that the vector operating on the physical (finite density) vacuum does not only create the vector meson (and γ-ray), but there are other many-body vector excitations, especially the Rapp/Wambach "rhosobar." The latter is an $N^\star(1520)$ excitation coupled to a nucleon hole, with vector quantum numbers. In fusing Brown/Rho and Rapp/Wambach [43] Rapp considered the production of rho mesons by pions, with the later decay of the ρ into dileptons in Fig. C.1. The $\pi\pi\rho$ coupling involves a $g_V^\star$ and the ρ-propagator, $(k^2 + m_\rho^{\star 2})^{-1}$. Thus, the vector dominance coupling at the photon point $k^2 = 0$ is given by

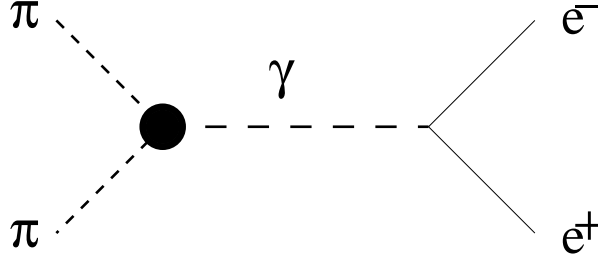$$g_{\rho\gamma}^\star = \frac{em_\rho^{\star 2}}{g_V^\star}, \tag{C.7}$$

36

Fig. C.2. Dilepton production via rhosobar dominance. The rhosobar is subsumed in the big dot on the left.

for gauge invariance. Now as we go away from the photon point to produce dileptons, in general the invariant mass of the latter is $\sim m_\rho^\star$, the $\vec{k}$ of the photon being small compared with the k_0. (Back-to-back kinematics involves only an $\sim 10\%$ error in dilepton production.) In doing this, we extend the vector dominance to the rhosobar, treated as a vector particle.

Thus, using vector dominance, Fig. C.1 reduces to Fig. C.2, where the ρ being subsumed in the big dot on the left.

We can check the validity of this approach at zero density and temperature from the particle-data book where $\Gamma_\rho = 150$ MeV and the branching ratio into $e^+ e^-$ is $(4.49 \pm 0.22) \times 10^{-5}$, or

$$\Gamma_{em} = 6.7 \text{ keV}. \tag{C.8}$$

In Fig. C.2 the branching ratio would be simply $\alpha^2 = 5.3 \times 10^{-5}$, a difference of only $\sim 10\%$.

We see from Tab. C.1 that $\Gamma_\rho^\star$ is large only in the last part of the mixed phase, from $T = 130$ to 120 MeV, the nearly hadron phase as the ρ-meson goes back on shell. We will call this the hadronic phase because the hadrons are nearly on shell. We assume the $\Gamma_{em} = \alpha^2 \Gamma^\star$, so that dileptons come only from the hadronic phase in CERES.

Rapp and Wambach[44] enhance the dileptons substantially by introducing the rhosobar, which at zero temperature puts $\sim 20\%$ of the ρ strength at 590 MeV. With temperature and many-body effects the width of the rhosobar broadens to $\Gamma \sim 250$ MeV, so that strength is moved down another ~ 125 MeV. With tails of strength functions, a substantial number of dileptons are produced down to ~ 250 MeV.

The factor $m_\rho^{\star 2}/g_V^\star$ must be introduced into the amplitude for dilepton production if Brown-Rho scaling is to be "fused" with Rapp/Wambach. As noted by Brown and Rho[43] this factor cancels the enhancement that would be obtained because of the B-R dropping ρ-mass, so that Rapp/Wambach should
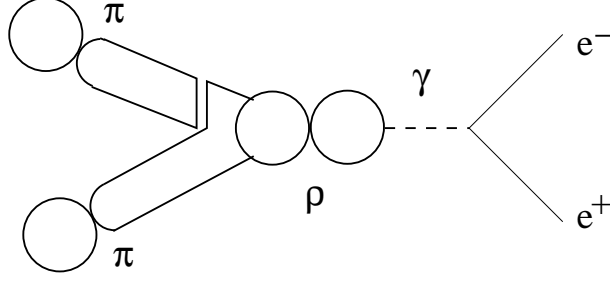
Fig. C.3. We redraw Fig. C.1 in the chirally restored region, $\pi + \pi \rightarrow \rho \rightarrow \gamma \rightarrow e^+ e^-$, in the one-bubble addition we used to obtain the $\Gamma = 380$ MeV of eq. (6).

fit well the hadronic dileptons.

We go into some detail with this because we believe that the hadronic dileptons in RHIC will be similar to those from the 200 GeV/nucleon CERES experiments. The scalar density in RHIC is not much less than at CERN because the antibaryons give a large contribution in the former. The factor of 2-4 in the ρ abundance at T_c will enhance the RHIC dileptons. However, the chirally restored region of temperatures will contribute at RHIC. In particular, the region of temperatures from T_c to $\sim 1.4 T_c$ will give dileptons in the range from 0 to ~ 400 MeV in our scenario, the broad region being given by the large width [6]. The contribution to the higher dilepton invariant masses from higher temperatures will be smaller because of the lower densities at the origin for these vibrations, this density relevant for dilepton production.

The phenomenological vector dominance implies that a strong interaction, in Fig. C.2 the ρ-meson hidden in the large black dot, precede a photon in the electromagnetic decay. Since, as noted in BLRS[3] the vector mesons move smoothly up through T_c, we believe that this assumption remains valid above T_c for the chirally restored mesons. In fact, the chirally restored ρ is the only vector particle above T_c, the rhosobar having disappeared with the nucleons. In any case, it seems the simplest way to couple dileptons to the ρ; namely, to add an off-shell γ-ray to the ρ (which is itself a sum of bubbles in the BLRS random phase description) as in Fig. C.1. This gives us the simple estimate for the chirally restored sector of

$$\Gamma_{em} \simeq \alpha^2 \Gamma_\rho \tag{C.9}$$

where we have estimated Γ_ρ to be ~ 380 MeV at $T_c + \epsilon$ in eq. (6).

One can see that eq. (C.9) is consistent with our calculation giving eq. (6) of Γ where we have added one bubble, Fig. 5, to each of the pions and ρ in fig. 3. We redraw the final π's and ρ in Fig. C.3, to recover the analog of Fig. C.1 in the chirally restored region.

We thus expect about equal dilepton abundances from the chirally restored

and hardonic sectors, with very little contribution from the mixed phase. The hadronic abundance should be about the same as in RHIC, and that from the chirally restored sector to be at lower energies, centered about $2m_\pi$, but with a large width which causes the upper end to overlap with the hadronic dileptons.

Because of the very strong coupling in the chirally restored sector for $T \sim T_c - 1.4T_c$, we cannot make any really quantitative calculations, but we believe our schematic model to give the main features. It will be exciting to confront them with experiment.

Of course, we know that the dilepton "cocktail" (background) is more than an order of magnitude greater in the region about dilepton invariant mass $\sim 2m_\pi$, so the low mass dileptons from the chirally restored sector may be difficult to separate from these, but hopefully there will be distinguishing characteristics.

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